

B.Sc. Semester-VI Examination, 2022-23**MATHEMATICS [Honours]**

Course ID : 62117 Course Code : SH/MTH/604/DSE-4

Course Title : Bio-Mathematics

Time : 2 Hours

Full Marks : 40

*The figures in the right-hand margin indicate marks.**Candidates are required to give their answers in their own words as far as practicable.**Notations and symbols have their usual meaning.*

1. Answer any **five** questions: 2×5=10
- What is the importance of Mathematical modelling?
 - What is 'crowding effect'?
 - Find the fixed points of the logistic map $x_{n+1} = 4x_n(1-x_n)$.
 - What is the difference between Immigration and Emigration in single species population model?
 - What are the forms of Holling's type II, III functional response?

- Write down the basic difference between exponential growth and logistic growth.
- Investigate the local stability of the steady states of the system

$$\frac{dx}{dt} = (x-1)(x-2)(x-3).$$

- What is Allee effect? Write down a single species Allee model.

2. Answer any **four** from the following questions:

5×4=20

- Write down the geometric interpretation of logistic growth model. 5
- Discuss the stability analysis of Prey-Predator model

$$\frac{dx}{dt} = ax - bxy; \quad \frac{dy}{dt} = -py + qxy$$
 where $a, b, p, q > 0$. 5
- Discuss the diffusion of glucose in the blood stream. 5
- What is the model for traffic on a highway and also discuss traffic wave propagation along a highway? 5
- Discuss Age Structured Population model and then deduce Leslie matrix. 5

[Turn Over]

f) What is Routh-Hurwitz criteria? Using this criterion show that

$$x' = dx - sy$$

$$y' = xy - rz$$

$$z' = (y - r)z$$

(d , s and r are the parameters of the above system), the system is unstable at non-trivial equilibrium point $(sr/d, r, sr/d)$. 5

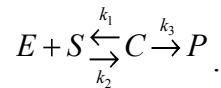
3. Answer any **one** of the following questions:

$$10 \times 1 = 10$$

a) Define Lotka-Volterra Competition model and discuss the stability analysis of this model.

$$4 + 6$$

b) i) Using the law of mass action write the system of ODEs for the following enzyme reaction:



ii) Formulate basic SIR model with demography and determine the prevalence of infection at equilibrium. 5+5=10

B.Sc. Semester-VI Examination, 2022-23

MATHEMATICS [Honours]

Course ID : 62117

Course Code : SH/MTH/604/DSE-4

Course Title : Mathematical Modelling

Time : 2 Hours

Full Marks : 40

The figures in the right-hand margin indicate marks. Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meaning.

1. Answer any **five** of the following questions:

$$2 \times 5 = 10$$

a) What is Mathematical Modelling?

b) Determine the radius of convergence of the power series

$$\sum_0^{\infty} \frac{x^n}{n!}$$

c) Find the singular point(s) of the ordinary of the differential equation

$$(x^2 - 4)y'' + xy' - y = 0.$$

d) Write down the Legendre's equation of order n .

e) Find the Laplace transform of $f(t) = t + \sin \omega t$.

- f) Show that $L^{-1}\left(\frac{s}{s^2 + \omega^2}\right) = \cos \omega t$, L^{-1} is the inverse Laplace operator.
- g) What is queuing model with single server queue?
- h) State the fundamental theorem of Linear Programming Problem (LPP).

2. Answer any **four** of the following questions:

$$5 \times 4 = 20$$

- a) Solve the IVP, $y'' - 3y' + 2y = e^{3t}$, with initial conditions $y(0) = 1$, $y'(0) = 0$, using Laplace transform.
- b) Consider the seed to be 14 and find five two digit random numbers using middle square method.
- c) In an infinite storage queue system with one server (M/M/1): (FCFS/) derive the following expressions:
- Expected number of customers in the queue,
 - Expected waiting time in the system.
- d) Solve the following LPP by simplex method:

$$\text{Maximize } z = 40x_1 + 30x_2$$

$$\text{subject to constraints: } x_1 + x_2 \leq 12,$$

$$2x_1 + x_2 \leq 16,$$

$$x_1, x_2 \geq 0.$$

- e) Find the power series solution of the equation

$$(1 - x^2)y'' + xy' - xy = 0$$

about $x = 0$.

- f) Find the inverse Laplace transform of

$$F(s) = \frac{2}{s^2 + 3s + 2}.$$

3. Answer any **one** of the following questions:

$$10 \times 1 = 10$$

- a) i) Solve the Bessel's equation of order zero by using the method of Frobenius and derive the Bessel function of the first kind of order 0.
- ii) If $a \equiv b \pmod{n}$ and $c \equiv d \pmod{n}$, then show that $a + c \equiv b + d \pmod{n}$ and $ac \equiv bd \pmod{n}$. (5+2)+3
- b) The optimal table of the following LPP

$$\text{Max } z = 4x_1 + 6x_2 + 2x_3$$

subject to constraints

$$x_1 + x_2 + x_3 \leq 3$$

$$x_1 + 4x_2 + 7x_3 \leq 9$$

$$x_1, x_2, x_3 \geq 0,$$

is

c_j			4	6	2	0	0
BV	C_B	X_B	x_1	x_2	x_3	x_4	x_5
x_1	4	1	1	0	-1	4/3	-1/3
x_2	6	2	0	1	2	-1/3	1/3
$c_j - z_j$			0	0	-6	-10/3	-2/3

- i) Discuss the effect of discrete change in the availability of resources from $[3 \ 9]^T$ to $[9 \ 6]^T$.
- ii) How much can $c_1 (= 4)$ be changed so that the optimality is not disturbed, and above table remains the optimal table? 6+4
