B.Sc. Semester-VI Examination, 2022-23

MATHEMATICS [Honours]

Course ID : 62117 Course Code : SH/MTH/604/DSE-4

Course Title : Bio-Mathematics

Time : 2 Hours Full Marks : 40

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meaning.

1. Answer any **five** questions: 2×5=10

   a) What is the importance of Mathematical modelling?

   b) What is 'crowding effect'?

   c) Find the fixed points of the logistic map $x_{n+1} = 4x_n(1-x_n)$.

   d) What is the difference between Immigration and Emigration in single species population model?

   e) What are the forms of Holling's type II, III functional response?

   f) Write down the basic difference between exponential growth and logistic growth.

   g) Investigate the local stability of the steady states of the system

$$\frac{dx}{dt} = (x-1)(x-2)(x-3).$$

   h) What is Allee effect? Write down a single species Allee model.

2. Answer any **four** from the following questions: 5×4=20

   a) Write down the geometric interpretation of logistic growth model. 5

   b) Discuss the stability analysis of Prey-Predator model

$\frac{dx}{dt} = ax - bxy;\ \frac{dy}{dt} = -py + qxy$ where $a, b, p, q > 0$. 5

   c) Discuss the diffusion of glucose in the blood stream. 5

   d) What is the model for traffic on a highway and also discuss traffic wave propagation along a highway? 5

   e) Discuss Age Structured Population model and then deduce Leslie matrix. 5

f) What is Routh-Hurwitz criteria? Using this criterion show that

$$x' = dx - sy$$

$$y' = xy - rz$$

$$z' = (y - r)z$$

(d, s and r are the parameters of the above system), the system is unstable at non-trivial equilibrium point $(sr/d,\ r,\ sr/d)$. 5

3. Answer any **one** of the following questions: 10×1=10

a) Define Lotka-Volterra Competition model and discuss the stability analysis of this model. 4+6

b) i) Using the law of mass action write the system of ODEs for the following enzyme reaction:

$$E + S \underset{k_2}{\overset{k_1}{\rightleftarrows}} C \xrightarrow{k_3} P .$$

ii) Formulate basic SIR model with demography and determine the prevalence of infection at equilibrium. 5+5=10

B.Sc. Semester-VI Examination, 2022-23

MATHEMATICS [Honours]

Course ID : 62117 Course Code : SH/MTH/604/DSE-4

Course Title : Mathematical Modelling

Time : 2 Hours Full Marks : 40

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meaning.

1. Answer any **five** of the following questions: 2×5=10

a) What is Mathematical Modelling?

b) Determine the radius of convergence of the power series

$$\sum_0^\infty \frac{x^n}{n!} .$$

c) Find the singular point(s) of the ordinary of the differential equation

$$(x^2 - 4)y'' + xy' - y = 0 .$$

d) Write down the Legendre's equation of order n.

e) Find the Laplace transform of $f(t) = t + \sin \omega t$.

f) Show that $L^{-1}\left(\frac{s}{s^2+\omega^2}\right)=\cos\omega t$, L^{-1} is the inverse Laplace operator.

g) What is queuing model with single server queue?

h) State the fundamental theorem of Linear Programming Problem (LPP).

2. Answer any **four** of the following questions: 5×4=20

a) Solve the IVP, $y''-3y'+2y=e^{3t}$, with initial conditions $y(0)$ = 1, $y'(0)=0$, using Laplace transform.

b) Consider the seed to be 14 and find five two digit random numbers using middle square method.

c) In an infinite storage queue system with one server (M/M/1): (FCFS/) derive the following expressions:

i) Expected number of customers in the queue,

ii) Expected waiting time in the system.

d) Solve the following LPP by simplex method:

Maximize $z=40x_1+30x_2$

subject to constraints: $x_1+x_2\le 12$,

$2x_1+x_2\le 16$,

$x_1, x_2\ge 0$.

e) Find the power series solution of the equation

$$(1-x^2)y''+xy'-xy=0$$

about x = 0.

f) Find the inverse Laplace transform of $F(s)=\frac{2}{s^2+3s+2}$.

3. Answer any **one** of the following questions: 10×1=10

a) i) Solve the Bessel's equation of order zero by using the method of Frobenius and derive the Bessel function of the first kind of order 0.

ii) If $a\equiv b\,(mod\ n)$ and $c\equiv d\,(mod\ n)$, then show that $a+c\equiv b+d\,(mod\ n)$ and $ac\equiv bd\,(mod\ n)$. (5+2)+3

b) The optimal table of the following LPP

Max $z=4x_1+6x_2+2x_3$

subject to constraints

$x_1+x_2+x_3\le 3$

$x_1+4x_2+7x_3\le 9$

$x_1, x_2, x_3\ge 0$,

is

c_j			4	6	2	0	0
BV	C_B	X_B	x_1	x_2	x_3	x_4	x_5
x_1	4	1	1	0	-1	4/3	$-1/3$
x_2	6	2	0	1	2	$-1/3$	1/3
$c_j - z_j$			0	0	-6	$-10/3$	$-2/3$

i) Discuss the effect of discrete change in the availability of resources from $[3\ 9]^T$ to $[9\ 6]^T$.

ii) How much can $c_1 (= 4)$ be changed so that the optimality is not disturbed, and above table remains the optimal table? 6+4

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